Since the morphodynamic time scale is T and the growth rate turns out to be of order 10^{-1} the e-folding time is of order

$$t_e^* \approx 10 \frac{(1-n) g^* \widetilde{D}_0^* (s-1) C^3}{8 U_0^{*2} 2\pi} T_{tide}^* \approx 100 - 1000 \text{ years}$$

Note that $T = S \dfrac{U_0^{*2}}{(1-n) g^* \widetilde{D}_0^*} \omega^* t^*$ $\qquad S = \dfrac{8}{(s-1) C^3}$

A REFINED APPROACH

1) Introduction of a tidal ellipse
2) Introduction of an angle between the bed shear stress and the depth averaged velocity
3) Analytical solution for a sinusoidal current
4) Sediment transport rate with a threshold value of θ
5) Finite amplitude sand banks

Hydrodynamics

Continuity equation

$$\frac{\partial D^*}{\partial t^*} + \frac{\partial (D^* U^*)}{\partial x^*} + \frac{\partial (D^* V^*)}{\partial y^*} = 0 \qquad D^* = \tilde{D}^* + \zeta^*$$

Momentum equations

$$\frac{\partial U^*}{\partial t^*} + U^* \frac{\partial U^*}{\partial x^*} + V^* \frac{\partial U^*}{\partial y^*} = -g^* \frac{\partial \zeta^*}{\partial x^*} - \frac{\tau_x^*}{\rho^* D^*} + f^* V^*$$

$$\frac{\partial V^*}{\partial t^*} + U^* \frac{\partial V^*}{\partial x^*} + V^* \frac{\partial V^*}{\partial y^*} = -g^* \frac{\partial \zeta^*}{\partial y^*} - \frac{\tau_y^*}{\rho^* D^*} - f^* U^*$$

where the Coriolis parameter is defined by $f^* = 2\Omega^* \sin\varphi$

THE DIMENSIONLESS PROBLEM

By introducing dimensionless variables using the tidal excursion (U_0^*/ω^*) as length scale, the inverse of the tide angular frequency (ω^{*-1}) as time scale, the maximum value (U_0^*) of the depth averaged velocity as velocity scale, the amplitude a^* of the tide to scale the free surface elevation and $\tilde{D}_0^*$ to scale the vertical coordinate, the problem reads:

$$a\frac{\partial D}{\partial t}+\frac{\partial(DU)}{\partial x}+\frac{\partial(DV)}{\partial y}=0 \quad \text{where} \quad D=\tilde{D}+a\zeta$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{a}\frac{\partial \zeta}{\partial x}-r\frac{\tau_x}{D}+fV$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{1}{a}\frac{\partial \zeta}{\partial y}-\frac{\tau_y}{D}-fU$$

The hydrodynamic problem is characterised by the following dimensionless parameters

$$a = \frac{a^*}{\tilde{D}_0^*} = \frac{U_0^*}{\sqrt{g^* \tilde{D}_0^*}} \qquad\qquad r = \frac{U_0^*}{\omega^* \tilde{D}_0^*}$$

where a^* is defined as function of U_0^*

Assuming a small amplitude tide, at the leading order of approximation the basic flow is described by

$$\frac{\partial U}{\partial t} = -\frac{1}{a}\frac{\partial \zeta}{\partial x} - r\frac{\tau_x}{D} + fV$$

The problem is balanced because for the basic flow the appropriate horizontal length scale is not U_0^* / ω^* but $L^* = \sqrt{g^* \widetilde{D}_0^*} / \omega^*$. Hence we should introduce $X = ax$ and

$$\frac{\partial U}{\partial t} = -\frac{\partial \zeta}{\partial X} - r\frac{\tau_x}{D} + fV$$

THE BED SHEAR STRESS

A relationship between the bottom shear stress and the depth averaged velocity should be introduced to close the problem. Different formulae can be adopted, e.g a linear relationship or a more complex constitutive relationship like

$$(\tau_x, \tau_y) = \frac{(U\cos\phi - V\sin\phi, U\sin\phi + V\cos\phi)\sqrt{U^2 + V^2}}{C^2}$$

which accounts for Coriolis effects which deviate the bed shear stress with respect to the depth averaged velocity (de Swart & Hulscher, 1995). The values of Φ can be obtained from 3D models. The resistance coefficient C, which depends on the bottom roughness z_r^*, can be evaluated by means of heuristic formulae, e.g. $C=5.75\log(11D_0^*/z^*_r)$

Morphodynamics

Sediment continuity equation

$$\frac{\partial h^*}{\partial t^*} + \frac{1}{(1-n)}\left[\frac{\partial Q_x^*}{\partial x^*} + \frac{\partial Q_y^*}{\partial y^*}\right] = 0$$

By using dimensionless variables

$$\frac{\partial h}{\partial T} = \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \qquad Q = \frac{Q^*}{\sqrt{(s-1)g^* d^*}}$$

where the morphodynamic time scale is introduced

$$T = \frac{t\,d}{(1-p)\,r\sqrt{\psi_d}} \qquad \text{with } \psi_d = \frac{(\omega \tilde{D}_0^*)^2}{(s-1)g^* d^*},\; d = \frac{d^*}{\tilde{D}_0^*}$$

THE SEDIMENT TRANSPORT RATE

Assuming that the suspended load is negligible, it is possible to identify two contributions : the bed load over an horizontal bed and the bed load induced by the bed slope : $\left(Q_x, Q_y\right) = \left(Q_x^{(b)}, Q_y^{(b)}\right) + \left(Q_x^{(p)}, Q_y^{(p)}\right)$

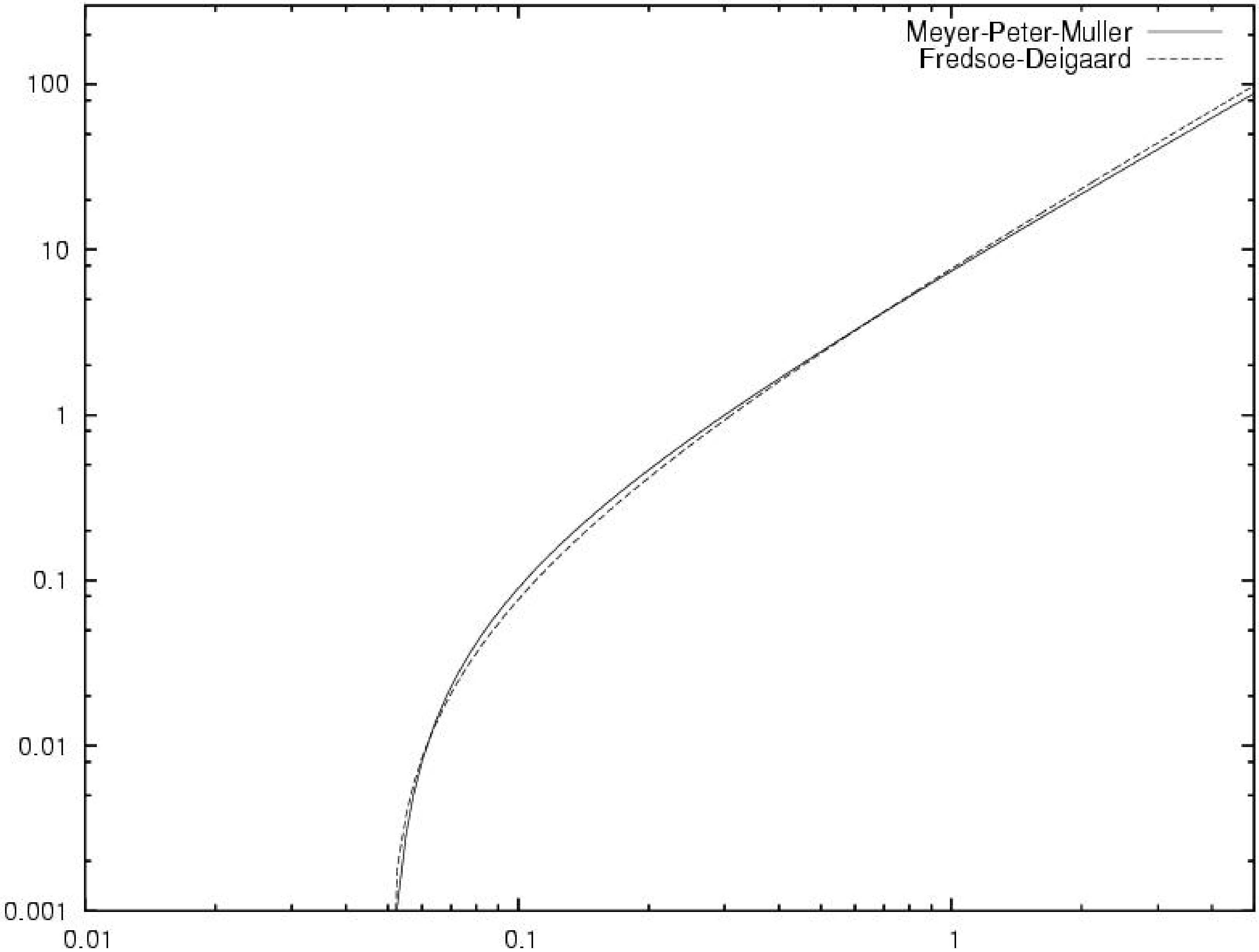
The bed load over a horizontal bed

(Fredsoe & Deigaard formula)

$$\left(Q_x^{(b)}, Q_y^{(b)}\right) = \frac{\left(\vartheta_x, \vartheta_y\right)}{\vartheta} \Phi(\vartheta)$$

$$\Phi(\vartheta) = \frac{30}{\pi\mu}\left(\vartheta - \vartheta_c\right)\left(\sqrt{\vartheta} - 0.7\sqrt{\vartheta_c}\right)$$

ϑ is the Shield parameter, ϑ_c is its critical value, $\mu \approx 1$



Sediment transport rate

Shields parameter

The down - slope sediment transport rate is

$$\left(Q_x^{(p)}, Q_y^{(p)}\right) = \frac{G}{r}\Phi\left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right)$$

$$G_{ss} = -1.7\frac{\vartheta_c}{\Phi}\frac{d\Phi}{d\vartheta}, \quad G_{nn} = -\frac{0.55}{\sqrt{\vartheta}}, \quad G_{sn} = G_{ns} = 0$$

$$\left(Q_x^{(p)}, Q_y^{(p)}\right) = \frac{\Phi}{r}$$

$$\left[\frac{\partial h}{\partial x}\left(G_{ss}\frac{\vartheta_x^2}{\vartheta^2} + G_{nn}\frac{\vartheta_y^2}{\vartheta^2}\right) + \frac{\partial h}{\partial y}\frac{\vartheta_x\vartheta_y}{\vartheta^2}\left(G_{ss} - G_{nn}\right);\right.$$

$$\left.\frac{\partial h}{\partial y}\left(G_{ss}\frac{\vartheta_y^2}{\vartheta^2} + G_{nn}\frac{\vartheta_x^2}{\vartheta^2}\right) + \frac{\partial h}{\partial x}\frac{\vartheta_x\vartheta_y}{\vartheta^2}\left(G_{ss} - G_{nn}\right)\right]$$

$$\nabla h = \left(\frac{\partial h}{\partial s}, \frac{\partial h}{\partial n} \right)$$

$$\frac{\partial h}{\partial s} = \frac{\nabla h \bullet \vartheta_0}{\left|\vartheta_0\right|} = \frac{1}{\left|\vartheta_0\right|}\left(\frac{\partial h}{\partial x}\vartheta_{0x} + \frac{\partial h}{\partial y}\vartheta_{0y} \right)$$

$$\overrightarrow{\frac{\partial h}{\partial s}} = \left(\frac{\partial h}{\partial x}\vartheta_{0x} + \frac{\partial h}{\partial y}\vartheta_{0y} \right)\frac{\left(\vartheta_{0x}, \vartheta_{0y}\right)}{\left|\vartheta_0\right|^2}$$

$$\overrightarrow{\frac{\partial h}{\partial n}} = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y} \right) - \overrightarrow{\frac{\partial h}{\partial s}} = \left[\frac{\partial h}{\partial x} - \left(\frac{\partial h}{\partial x}\vartheta_{0x} + \frac{\partial h}{\partial y}\vartheta_{0y} \right)\frac{\vartheta_{0x}}{\left|\vartheta_0\right|^2}, \right.$$

$$\left. \frac{\partial h}{\partial y} - \left(\frac{\partial h}{\partial x}\vartheta_{0x} + \frac{\partial h}{\partial y}\vartheta_{0y} \right)\frac{\vartheta_{0y}}{\left|\vartheta_0\right|^2} \right]$$

THE TIME DEVELOPMENT OF A SMALL BOTTOM PERTURBATION

Let us consider a bottom perturbation of the flat bottom

$$h = \text{cost} + \varepsilon h_1 = \text{cost} + \varepsilon\left[\Pi_1 e^{i(\alpha_x x + \alpha_y y)} + c.c.\right]$$

since $a << 1$, $\tilde{D} = D$

$D = D_0 + \varepsilon D_1$ with $D_0 = 1, \quad D_1 = -\Pi_1 E, \quad E = e^{i(\alpha_x x + \alpha_y y)}$

Moreover,

$U = U_0 + \varepsilon U_1 \qquad\qquad V = V_0 + \varepsilon V_1$

$\zeta = \zeta_0 + \varepsilon a \zeta_1$

$\left(\tau_x, \tau_y\right) = \left(\tau_{x0}, \tau_{y0}\right) + \varepsilon\left(\tau_{x1}, \tau_{y1}\right)$

THE BASIC STATE (flat bottom)

We simulate the M2 constituent with a given tidal ellipse

$$U_0 = e^{it} + c.c.$$

$$V_0 = \frac{b}{a} e^{it} + c.c.$$

The tidal ellipse is forced by appropriate surface slopes

$$\frac{\partial \zeta_0}{\partial X} = -\frac{\partial U_0}{\partial t} - r\frac{\tau_{x0}}{Y_0} - fV_0$$

$$\frac{\partial \zeta_0}{\partial Y} = -\frac{\partial V_0}{\partial t} - r\frac{\tau_{y0}}{Y_0} - fU_0$$

where

;

$X = ax, \qquad Y = ay$

THE PERTURBATION PROBLEM

By plugging the expansions of $U, V, \ldots$ into continuity and momentum equations, we have

$$\left[\frac{\partial U_1}{\partial x} + \frac{\partial V_1}{\partial y}\right] + U_0 \frac{\partial D_1}{\partial x} + V_0 \frac{\partial D_1}{\partial y} = 0$$

$$\frac{\partial U_1}{\partial t} + U_0 \frac{\partial U_1}{\partial x} + V_0 \frac{\partial U_1}{\partial y} = -\frac{\partial \zeta_1}{\partial x} - r\left[\tau_{x0} + \tau_{x1}\right] + fV_1$$

$$\frac{\partial V_1}{\partial t} + U_0 \frac{\partial V_1}{\partial x} + V_0 \frac{\partial V_1}{\partial y} = -\frac{\partial \zeta_1}{\partial y} - r\left[\tau_{y0} + \tau_{y1}\right] - fU_1$$

If we write

$$U_1 = \Pi_1 \hat{U}_1 E, \qquad V_1 = \Pi_1 \hat{V}_1 E, \ldots$$

such that $\hat{U}_1, \hat{V}_1, \ldots$ depend only on time, the bed shear stress expansions can be easily worked out

$$\left(\tau_{x0}, \tau_{y0}\right) = T_0 R_0 \left(U_0, V_0\right)$$

$$\left(\tau_{x1}, \tau_{y1}\right) = \Pi_1 \left[\left(\hat{U}_1, \hat{V}_1\right) T_0 R_0 + \left(U_0, V_0\right) \left(\hat{T}_1 R_0 + T_0 \frac{\hat{U}_1 U_0 + \hat{V}_1 V_0}{R_0} \right) \right] E$$

where

$$T_0 = \frac{1}{C_0^2}, \qquad R_0 = \sqrt{U_0^2 + V_0^2}, \qquad \hat{T}_1 = \frac{5}{C_0^5}$$

Combining the two momentum equations we obtain the equation for the vorticity $\hat{\eta}_1 = i\alpha_x \hat{V}_1 - i\alpha_y \hat{U}_1$

$$\frac{d\hat{\eta}_1}{dt} + i\left(\alpha_x U_0 + \alpha_y V_0\right)\hat{\eta}_1 = -if\left(\alpha_x \hat{U}_1 + \alpha_y \hat{V}_1\right) - r\left[-i\alpha_y \tau_{x0} + i\alpha_x \tau_{y0}\right.$$

$$\left. + \left(R_0 \hat{T}_1 \left(U_0 \hat{U}_1 + V_0 \hat{V}_1\right) + \frac{T_0}{R_0}\right)\left(-i\alpha_y U_0 + i\alpha_x V_0\right) + R_0 T_0 \hat{\eta}_1 \right]$$

which can be written in the form

$$\frac{d\hat{\eta}_1}{dt} + F(t)\hat{\eta}_1 = G(t)$$

The solution can be obtained by expanding $\hat{\eta}_1$ as a Fourier series of time $\left(\hat{\eta}_1 = \sum_{n=-\infty}^{\infty} \hat{\eta}_{1,n} e^{in\,t}\right)$ and solving an algebraic linear system for the unknowns $\hat{\eta}_{1,n}$

Continuity equation

$$\alpha_x \hat{U}_1 + \alpha_y \hat{V}_1 = \alpha_x \hat{U}_0 + \alpha_y \hat{V}_0$$

along with vorticity definition

$$\hat{\eta}_1 = i\alpha_x \hat{V}_1 - i\alpha_y \hat{U}_1$$

allow the two velocity components to be determined

$$\hat{U}_1 = \frac{\alpha_x\left(\alpha_x \hat{U}_0 + \alpha_y \hat{V}_0\right)}{\alpha_x^2 + \alpha_y^2} + \frac{i\alpha_y}{\alpha_x^2 + \alpha_y^2}\hat{\eta}_1$$

$$\hat{V}_1 = \frac{\alpha_y\left(\alpha_x \hat{U}_0 + \alpha_y \hat{V}_0\right)}{\alpha_x^2 + \alpha_y^2} - \frac{i\alpha_x}{\alpha_x^2 + \alpha_y^2}\hat{\eta}_1$$

Because of the presence of the small parameter ε it is also possible to write

$$\left(Q_x^{(b)}, Q_y^{(b)}\right) = \left(Q_{x0}^{(b)}, Q_{y0}^{(b)}\right) + \varepsilon\left(Q_{x1}^{(b)}, Q_{y1}^{(b)}\right)$$

$$\Phi(\vartheta) = \Phi_0 + \varepsilon\Phi_1;$$

$$\Phi_0 = \frac{30}{\pi\mu}\left(\vartheta_0 - \vartheta_c\right)\left(\sqrt{\vartheta_0} - 0.7\sqrt{\vartheta_c}\right)$$

$$\Phi_1 = \frac{30}{\pi\mu}\vartheta_1 \frac{3\vartheta_0 - \vartheta_c - 1.4\sqrt{\vartheta_0\vartheta_c}}{2\sqrt{\vartheta_0}}$$

$\left(Q_{x0}^{(b)}, Q_{y0}^{(b)}\right) = \ldots$ (these functions are very long

$\left(Q_{x1}^{(b)}, Q_{y1}^{(b)}\right) = \ldots$ but simple to be obtained)

$$\left(Q_{x0}^{(p)}, Q_{y0}^{(p)}\right) = (0,0)$$

$$\left(Q_{x1}^{(p)}, Q_{y1}^{(p)}\right) = \frac{\Phi_0}{r}$$

$$\left[\frac{\partial h_1}{\partial x}\left(G_{ss0}\frac{\vartheta_{x0}^2}{\vartheta_0{}^2} + G_{nn0}\frac{\vartheta_{y0}^2}{\vartheta_0{}^2}\right) + \frac{\partial h_1}{\partial y}\frac{\vartheta_{x0}\vartheta_{y0}}{\vartheta_0{}^2}\left(G_{ss0} - G_{nn0}\right);\right.$$

$$\left.\frac{\partial h_1}{\partial y}\left(G_{ss0}\frac{\vartheta_{y0}^2}{\vartheta_0{}^2} + G_{nn0}\frac{\vartheta_{x0}^2}{\vartheta_0{}^2}\right) + \frac{\partial h_1}{\partial x}\frac{\vartheta_{x0}\vartheta_{y0}}{\vartheta_0{}^2}\left(G_{ss0} - G_{nn0}\right)\right]$$

If the bottom perturbation $h = \text{cost} + \varepsilon\left[\Pi(T)\, e^{i(\alpha_x x + \alpha_y y)} + c.c.\right]$ along with the expansion of the sediment transport rate are considered and the notation $\left(Q_{x1}, Q_{y1}\right) = \Pi\left(\hat{Q}_{x1}, \hat{Q}_{y1}\right)E$ is employed, the sediment continuity equation at order ε leads to

$$\frac{\partial \Pi}{\partial T} = -i\left[\alpha_x \hat{Q}_{x1} + \alpha_y \hat{Q}_{y1}\right]\Pi = \Gamma(t)\Pi$$

From

$$\frac{d\Pi}{dT} = \Gamma(t)\Pi$$

it follows

$$\Pi(T) = \Pi_0 \exp\left(\int \Gamma(t)dt\right)$$

The averaged growth rate $\overline{\Gamma} = \int_0^{2\pi} \Gamma(t)dt$

Real part $\overline{\Gamma}_R$ ⇨ Growth rate

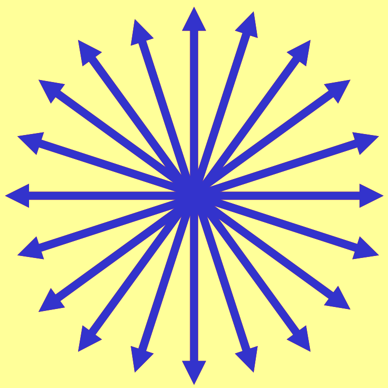
Imaginary part $\overline{\Gamma}_I$ ⇨ Migration speed of the perturbation

STUDY CASES

UNIDIRECTIONAL TIDES (e=0):

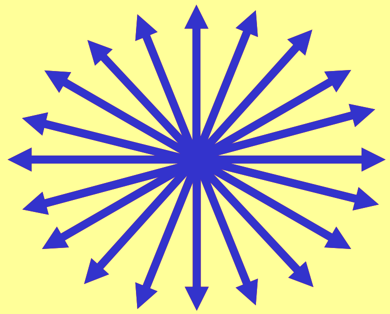
LOW ELLIPTICITY TIDES:

“Sand banks are likely to occur where the tidal currents are rotary or have low ellipticity”
Dyer & Huntley, Est. Coastal Shelf Science, 1999



e=-1

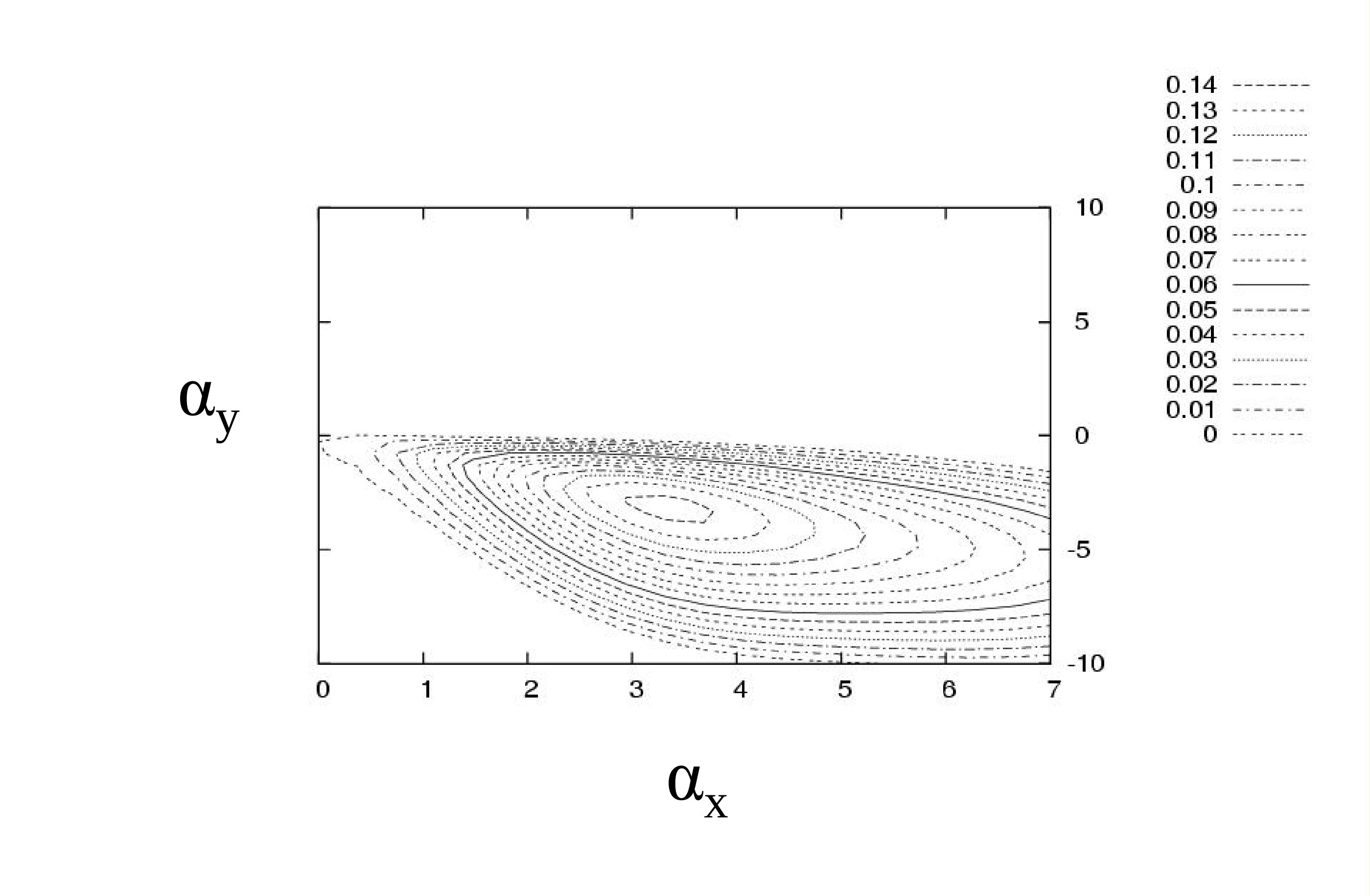
clock-wise rotating velocity vector



e=0.8

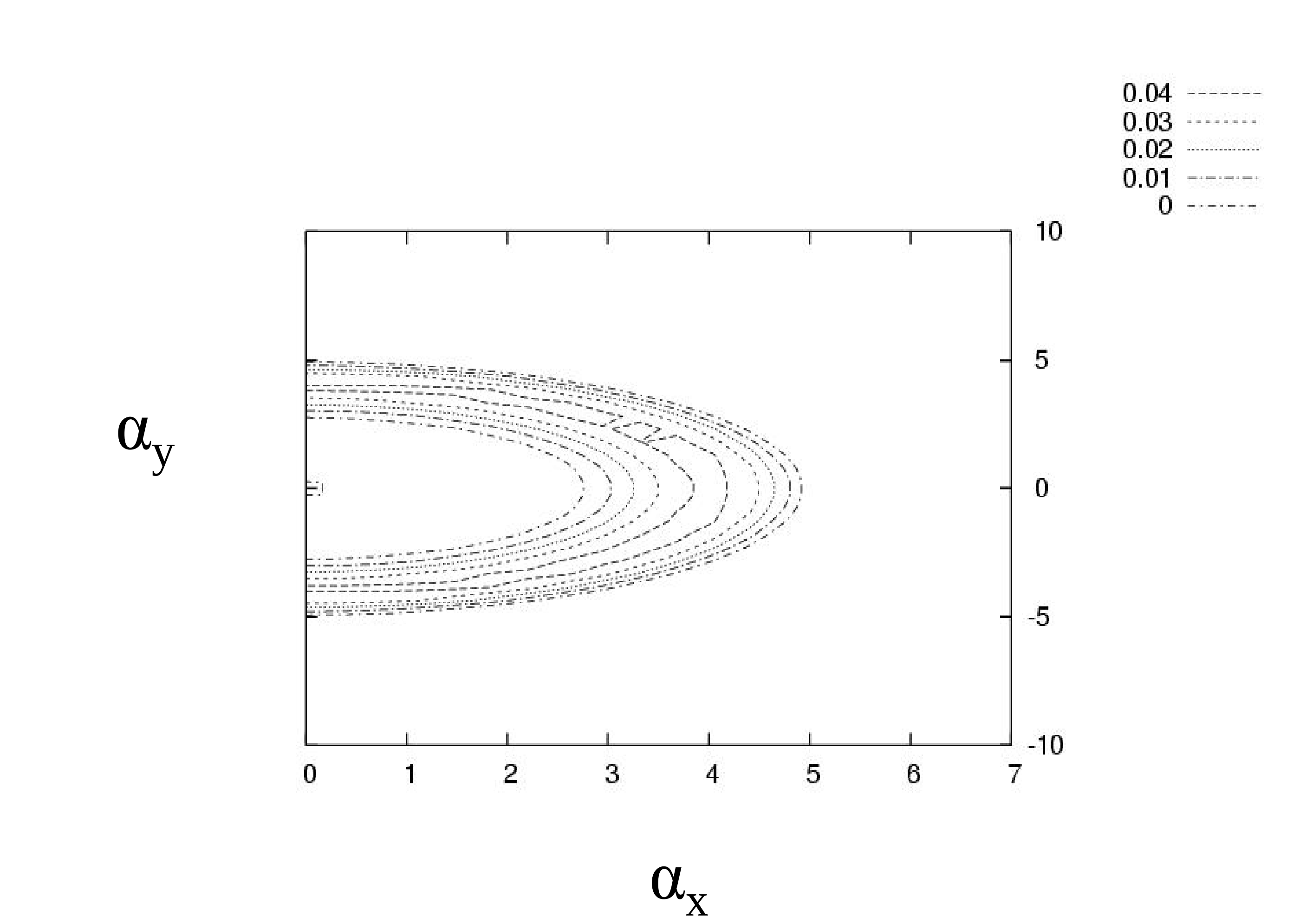
counterclock-wise rotating velocity vector

Growth rate versus the wavenumbers of the bottom perturbation (the x-axis is aligned with the major axis of the tidal ellipse)



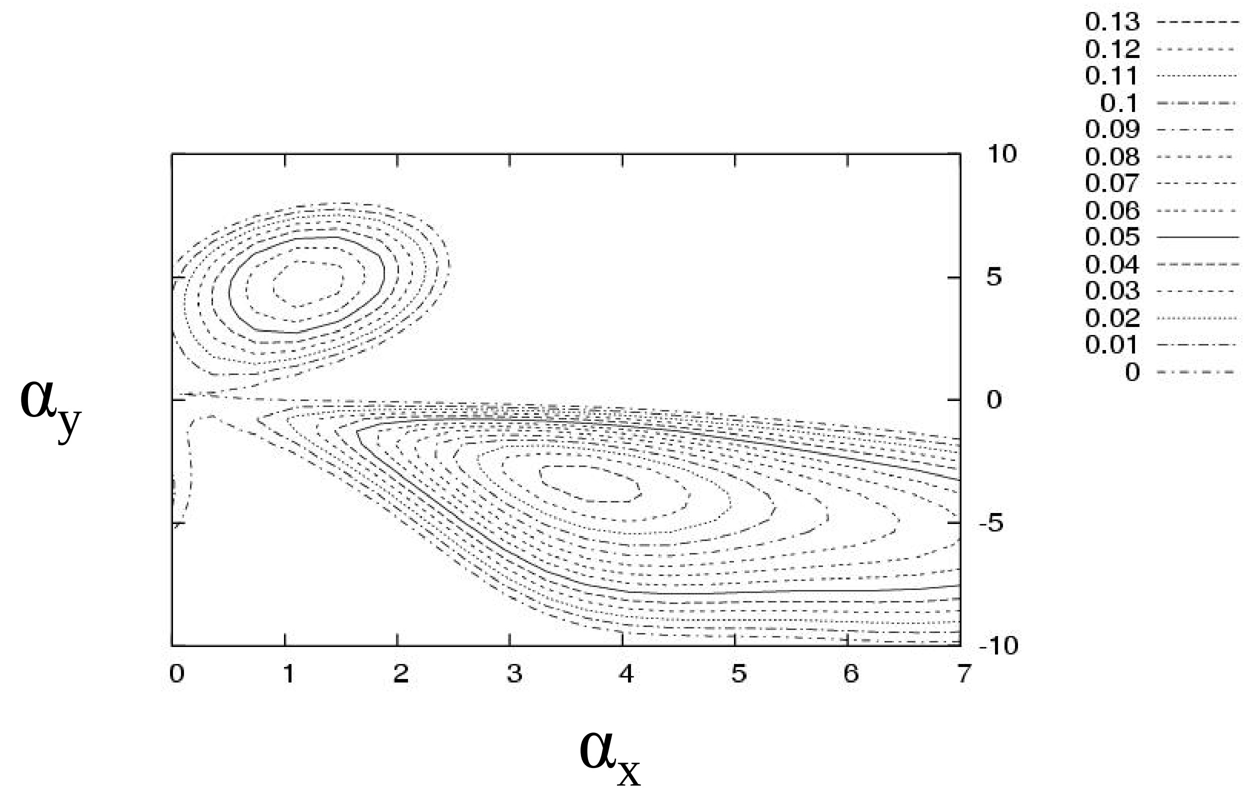
f=0.8, r = 120, z_r=0.001, a/b=0, Ψ_d=0.0058

Growth rate versus the wavenumbers of the bottom perturbation (the x-axis is aligned with the major axis of the tidal ellipse)



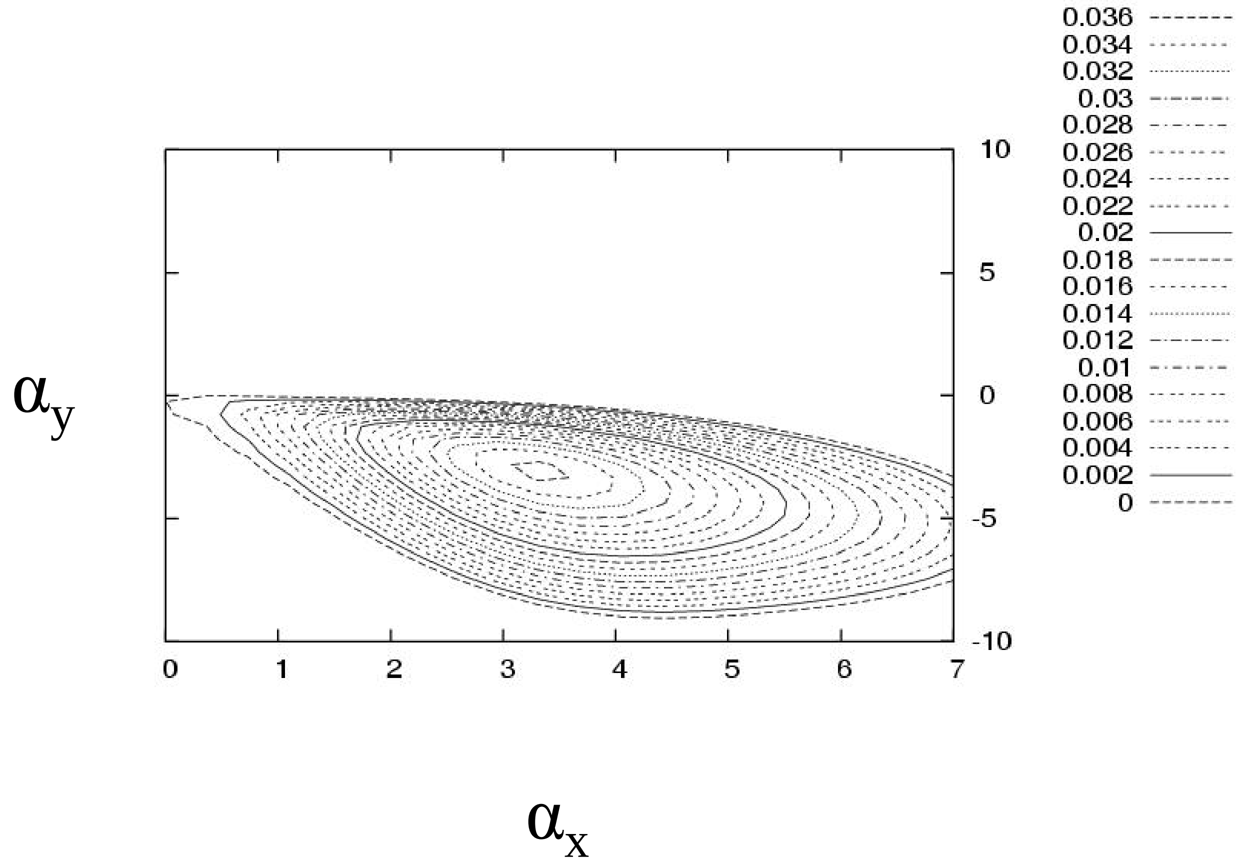
f=0.8, r = 120, z_r=0.001, a/b=1, Ψ_d=0.0058

Growth rate versus the wavenumbers of the bottom perturbation (the x-axis is aligned with the major axis of the tidal ellipse)



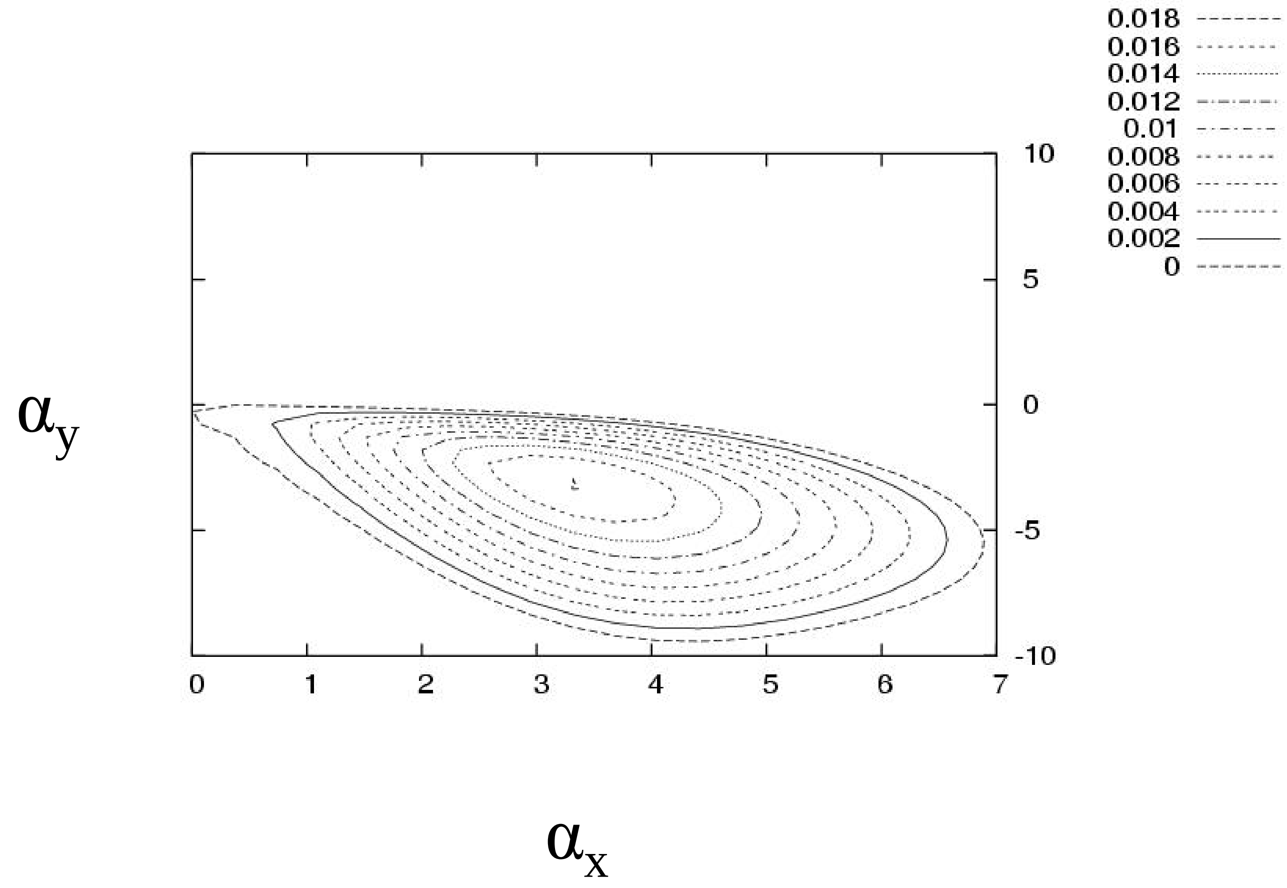
f=0.8, r = 120, z_r=0.001, a/b=0.2, Ψ_d=0.0058

Growth rate versus the wavenumbers of the bottom perturbation (the x-axis is aligned with the major axis of the tidal ellipse)



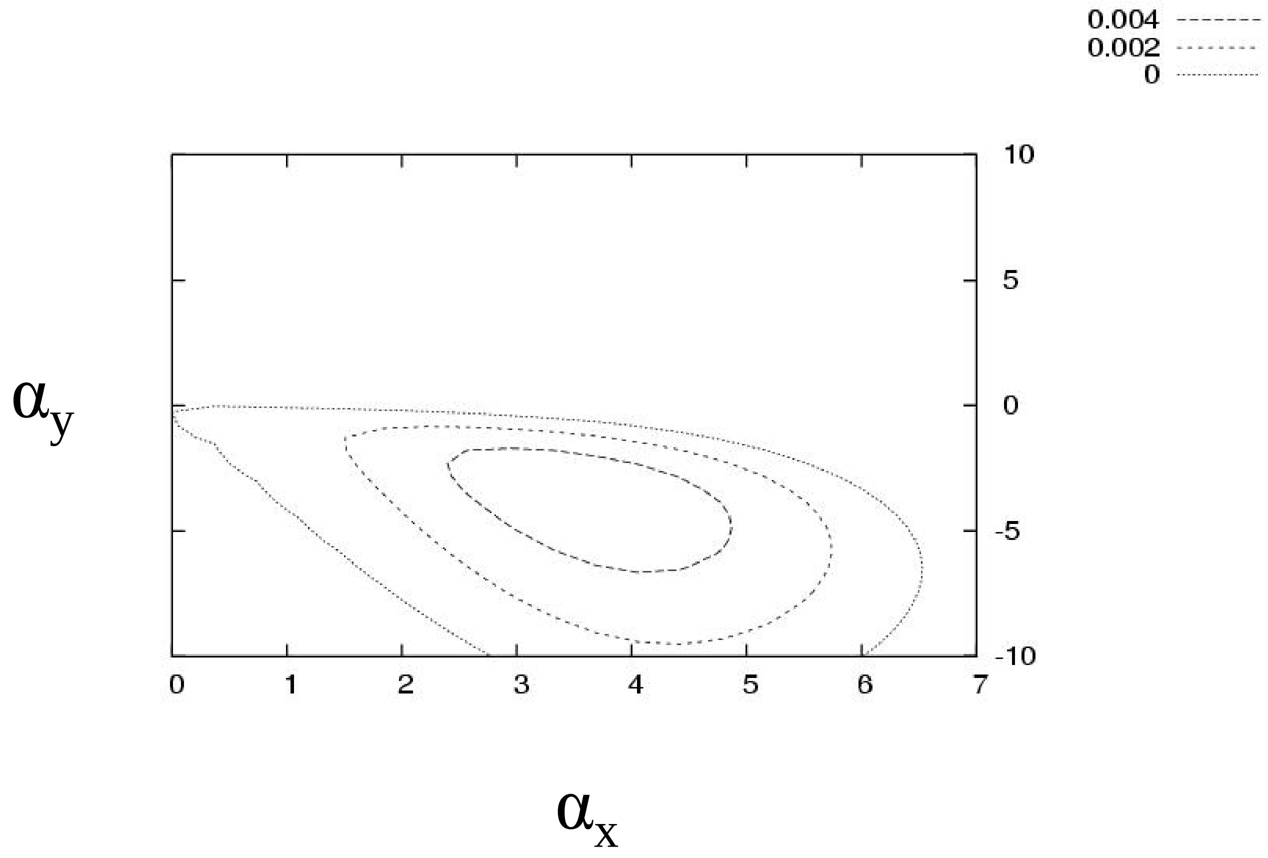
f=0.8, *r* = 90, z_r=0.001, *a*/*b*=0.2, Ψ_d=0.0058

Growth rate versus the wavenumbers of the bottom perturbation (the x-axis is aligned with the major axis of the tidal ellipse)



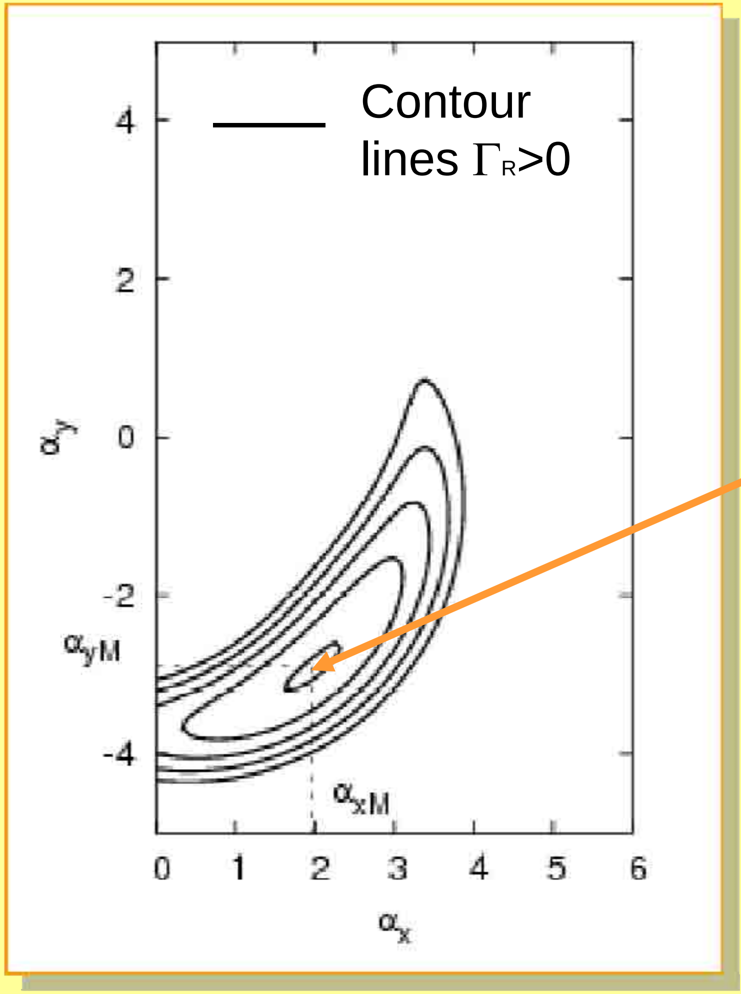
f=0.8, r = 80, z_r=0.001, a/b=0.2, Ψ_d=0.0058

Growth rate versus the wavenumbers of the bottom perturbation (the x-axis is aligned with the major axis of the tidal ellipse)



f=0.8, r = 70, z_r=0.001, a/b=0.2, Ψ_d=0.0058

Semidiurnal tide U^*_0=0.55 m/s $\tilde{D}^*_0$= 30 m e= 0.9 (clockwise rotating)
d*=0.2 mm z^*_r=3 cm ϕ=7.5°



Largest growth rate: counter clockwise rotated sandbanks with wavelength equal to 7 km

Growth rate as a function of the wavenumbers

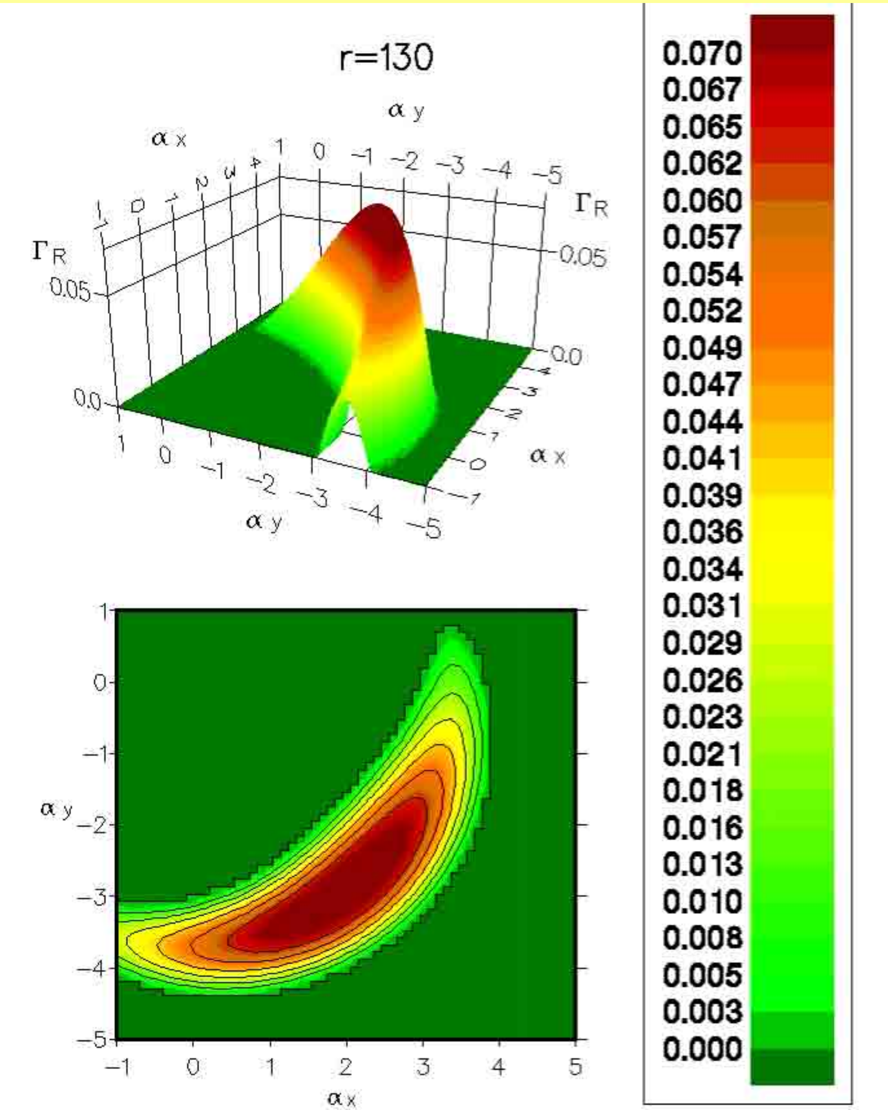
SAND BANKS OF FINITE AMPLITUDE

Semidiurnal tide U^*_0=0.55 m/s $\tilde{D}^*_0$= 30 m e= 0.9 (clockwise rotating)
d^*=0.2 mm z^*_r=3 cm ϕ=7.5°

As *r* tends to a critical value, ranging around 93, the growth rate decreases to 0.

Although the most unstable wave number decreases as r tends to its critical value, its limit value is still finite!!

It is possible to perform a weakly non linear stability analysis !!!



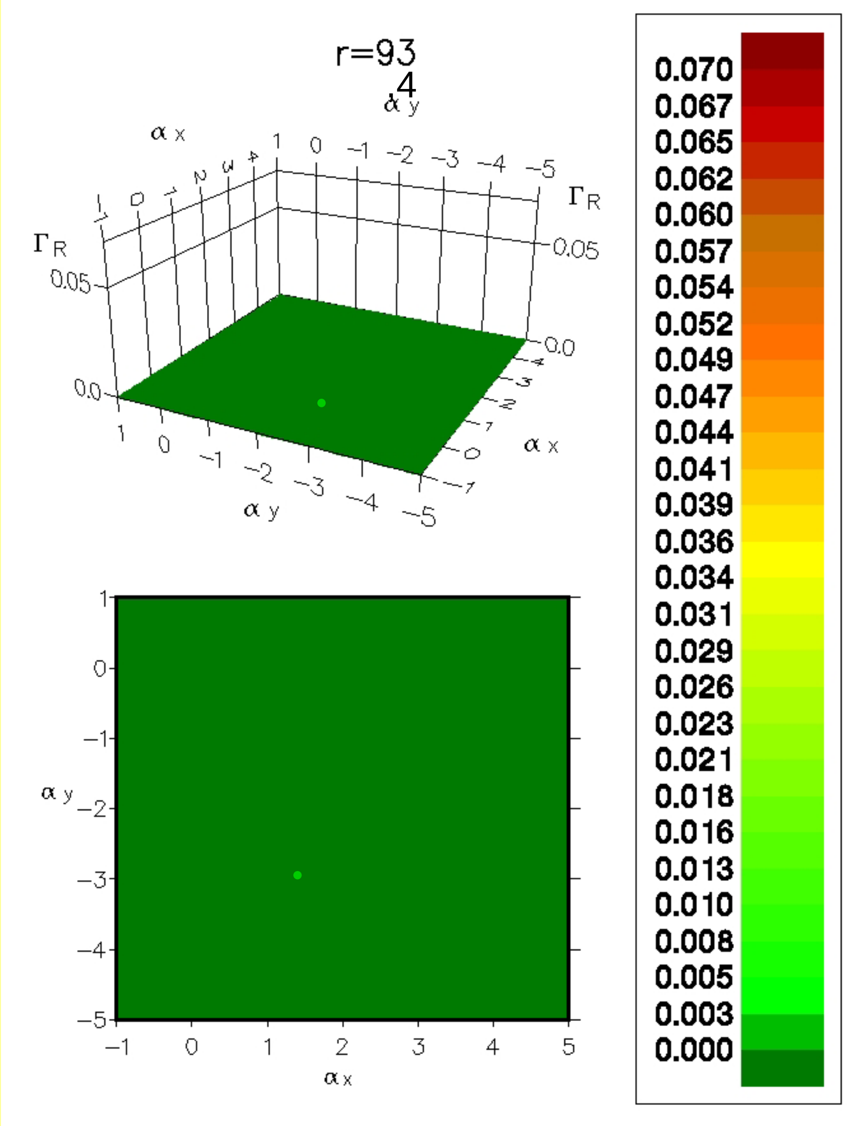
SAND BANKS OF FINITE AMPLITUDE

Semidiurnal tide U^*_0=0.40 m/s h^*_0= 30 m e= 0.9 (clockwise rotating)
d^*_s=0.2 mm z^*_r=3 cm ϕ=7.5°

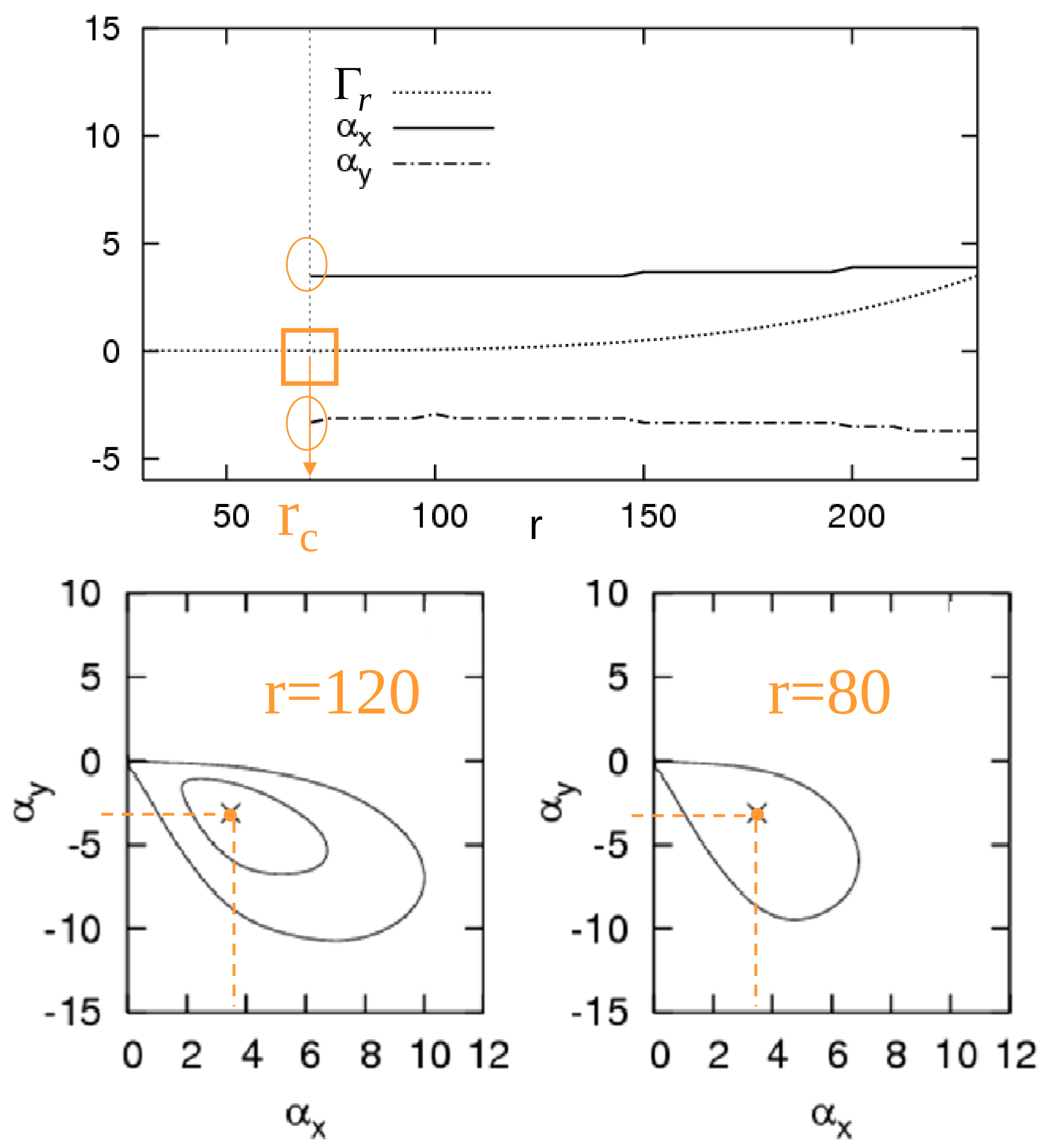
As *r* tends to a critical value, ranging around 93, the growth rate decreases to 0.

Although the most unstable wave number decreases as r tends to its critical value, its limit value is still finite!!

It is possible to perform a weakly non linear stability analysis !!!



SAND BANKS OF FINITE AMPLITUDE



PARAMETERS

e=0

Rp=11.4

ψ_d=0.00551

WEAKLY NON LINEAR ANALYSIS

- * $r = r_c + \varepsilon\, r_1$ Cose to critical conditions
- * $\mathcal{T} = \varepsilon T$ Slow time scale

NON LINEAR INTERACTIONS

Order $(\varepsilon^{\beta})^0$ → 1

Order $(\varepsilon^{\beta})^1$ → $\Pi(\mathcal{T})\, e^{i(\alpha_x x + \alpha_y y)} + c.c.$

Order $(\varepsilon^{\beta})^2$ → $\Pi^2(\mathcal{T})\, e^{2i(\alpha_x x + \alpha_y y)} + c.c.$ Second harmonic; $\left|\Pi(\mathcal{T})\right|^2$ Basic state distorsion

Order $(\varepsilon^{\beta})^3$ → $\Pi^3(\mathcal{T})\, e^{3i(\alpha_x x + \alpha_y y)} + c.c.$ $\quad \Pi(\mathcal{T})\left|\Pi(\mathcal{T})\right|^2 e^{i(\alpha_x x + \alpha_y y)} + c.c.$

The term $\varepsilon^{3\beta}\Pi(\mathcal{T})\left|\Pi(\mathcal{T})\right|^2$ Should be balanced by

$$\frac{\partial}{\partial T}(\varepsilon^{\beta}\Pi(\mathcal{T})) = \varepsilon^{\beta+1}\frac{d\Pi(\mathcal{T})}{d\mathcal{T}}$$

→ $\beta = 1/2$

WEAKLY NON LINEAR ANALYSIS

BOTTOM PERTURBATION

Ampiezza del modo più instabile

Amplitude of the first superharmonic component

legato a $\longrightarrow$ $\Pi_1^{\,2}$

$$h = 1 - \varepsilon^{\frac{1}{2}}\left[\Pi_1(\mathcal{T})\, e^{i(\alpha_x x + \alpha_y y)} + c.c.\right] - \varepsilon\left[\Pi_2(\mathcal{T})\, e^{2i(\alpha_x x + \alpha_y y)} + c.c.\right] +$$

$$- \varepsilon^{\frac{3}{2}}\left[\Pi_3(\mathcal{T})\, e^{i(\alpha_x x + \alpha_y y)} + c.c. + \ldots\right]$$

Average over a tidal cycle is vanishing

Principio di conservazione della massa per la fase solida $O(\varepsilon^{3/2})$ ⇒

$$\frac{d\Pi_1}{d\mathcal{T}} + \frac{d\Pi_3}{d\mathcal{T}} = a_1\Pi_1 + a_2\left|\Pi_1\right|^2\Pi_1 + \gamma\Pi_3$$

Equazione Landau-Stuart

⇓

$$\left|\Pi_1\left(\mathcal{T}\right)\right| = \left[\frac{\mathcal{Re}(a_1)}{\exp[-2\mathcal{Re}(a_1)\mathcal{T}] - \mathcal{Re}(a_2)}\right]^{1/2}$$

For T → ∞

$$\left|\Pi_{1e}\right| = \left[-\frac{\mathcal{Re}(a_1)}{\mathcal{Re}(a_2)}\right]^{1/2}$$

EQ. AMP

WEAKLY NON LINEAR ANALYSIS

BOTTOM PERTURBATION

Most unstable mode amplitude

Amplitude of the first superharmonic component

Linked to → $\Pi_1^{\,2}$

$$h = 1 - \varepsilon^{\frac{1}{2}}\left[\Pi_1(\mathcal{T})\, e^{i(\alpha_x x + \alpha_y y)} + c.c.\right] - \varepsilon\left[\Pi_2(\mathcal{T})\, e^{2i(\alpha_x x + \alpha_y y)} + c.c.\right] +$$

$$- \varepsilon^{\frac{3}{2}}\left[\Pi_3(\mathcal{T})\, e^{i(\alpha_x x + \alpha_y y)} + c.c. + \ldots\right]$$

Average over a tidal cycle is vanishing

Solid mass conservation for $O(e^{3/2})$ ⇨

$$\frac{d\Pi_1}{d\mathcal{T}} + \frac{d\Pi_3}{d\mathcal{T}} = a_1\Pi_1 + a_2\left|\Pi_1\right|^2\Pi_1 + \gamma\Pi_3$$

Equazione Landau-Stuart

⇩

$$\left|\Pi_1\left(\mathcal{T}\right)\right| = \left[\frac{\mathcal{Re}(a_1)}{\exp[-2\mathcal{Re}(a_1)\mathcal{T}] - \mathcal{Re}(a_2)}\right]^{1/2}$$

For T → ∞

$$\left|\Pi_{1e}\right| = \left[-\frac{\mathcal{Re}(a_1)}{\mathcal{Re}(a_2)}\right]^{1/2}$$

EQ. AMP

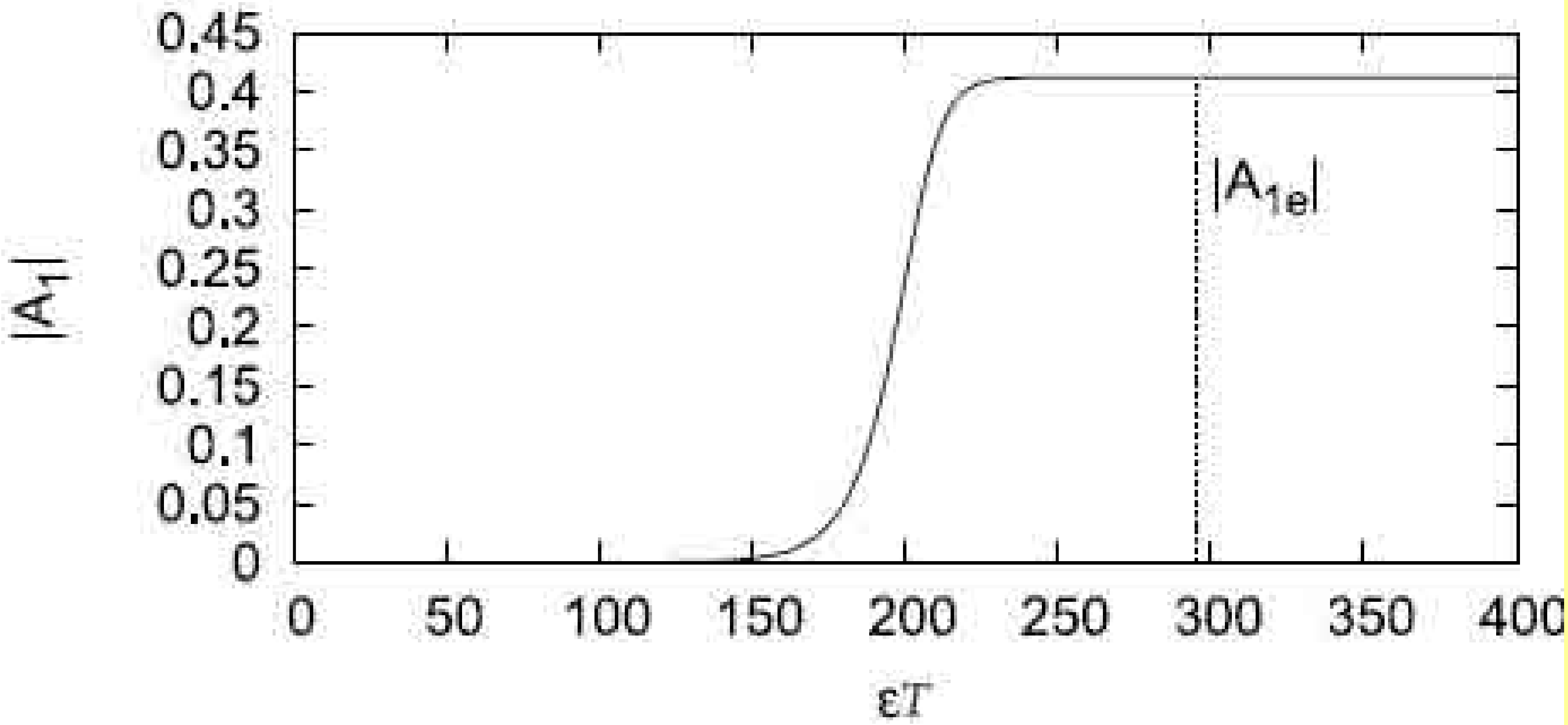


Figure 5. Time development of the fastest growing mode amplitude $|A_1|$ predicted on the basis of the Landau-Stuart equation (46) for $r = 85$, $e = -0.9$, $z_r = 6 \times 10^{-4}$, $f = 0.8$, $\varphi = 7.5°$, $d = 4 \times 10^{-6}$, $\psi_p = 1.52 \times 10^{-2}$, $\epsilon = 0.1$. The temporal scale used in the plot is the slow time scale associated with the growth of the perturbations $\mathcal{T} = \epsilon T$.

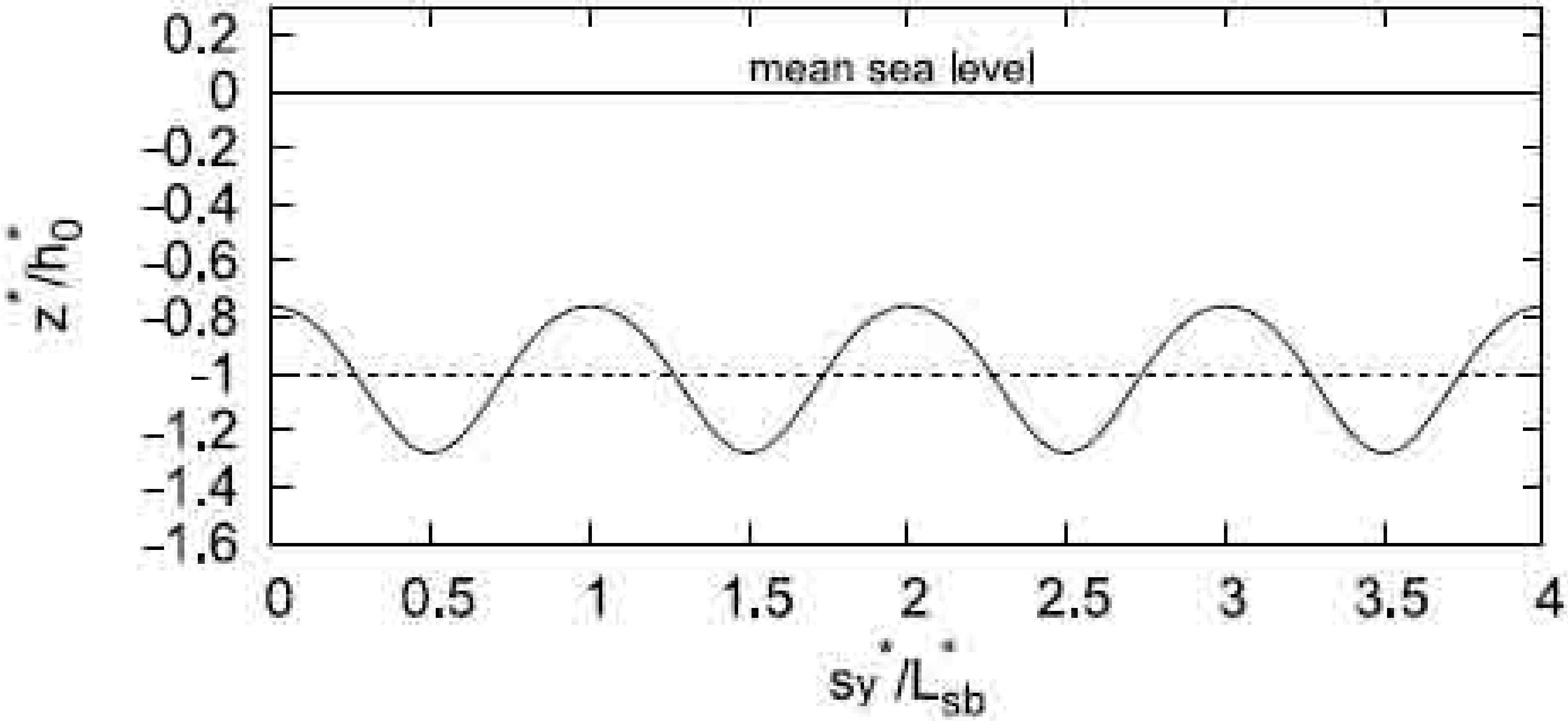
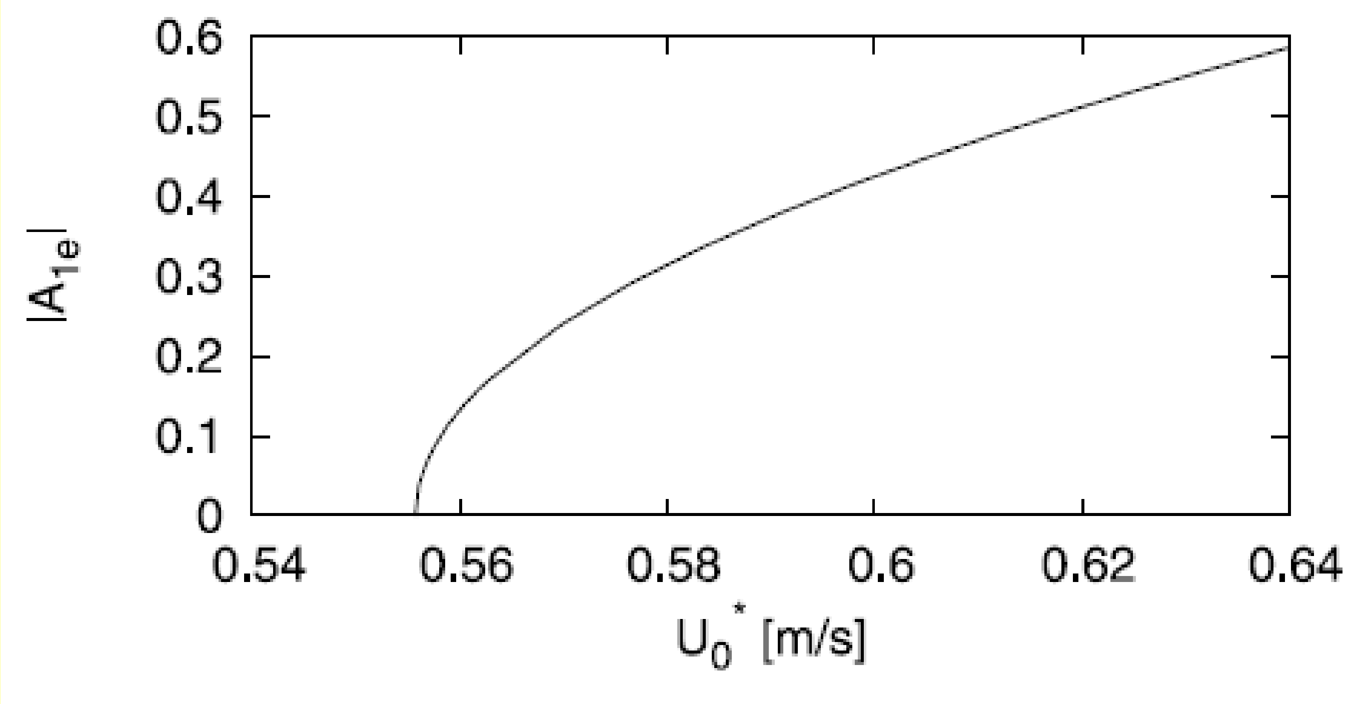
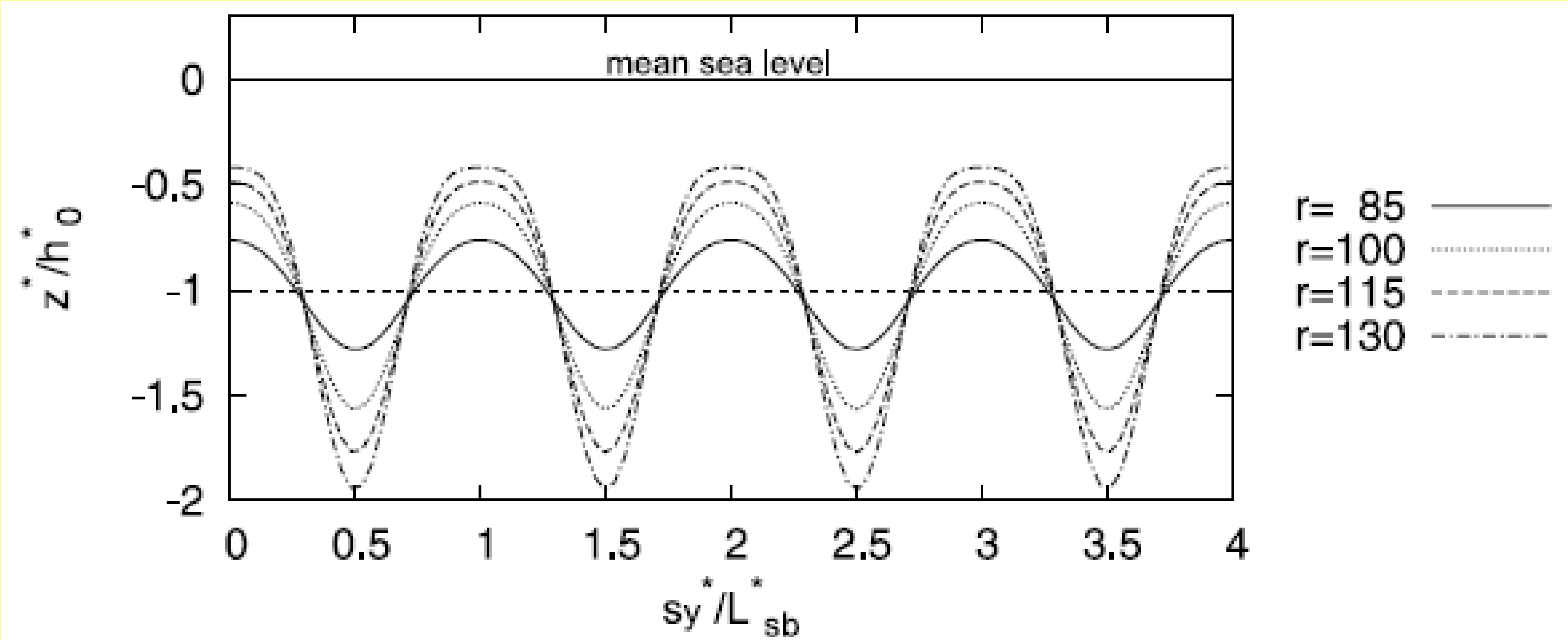


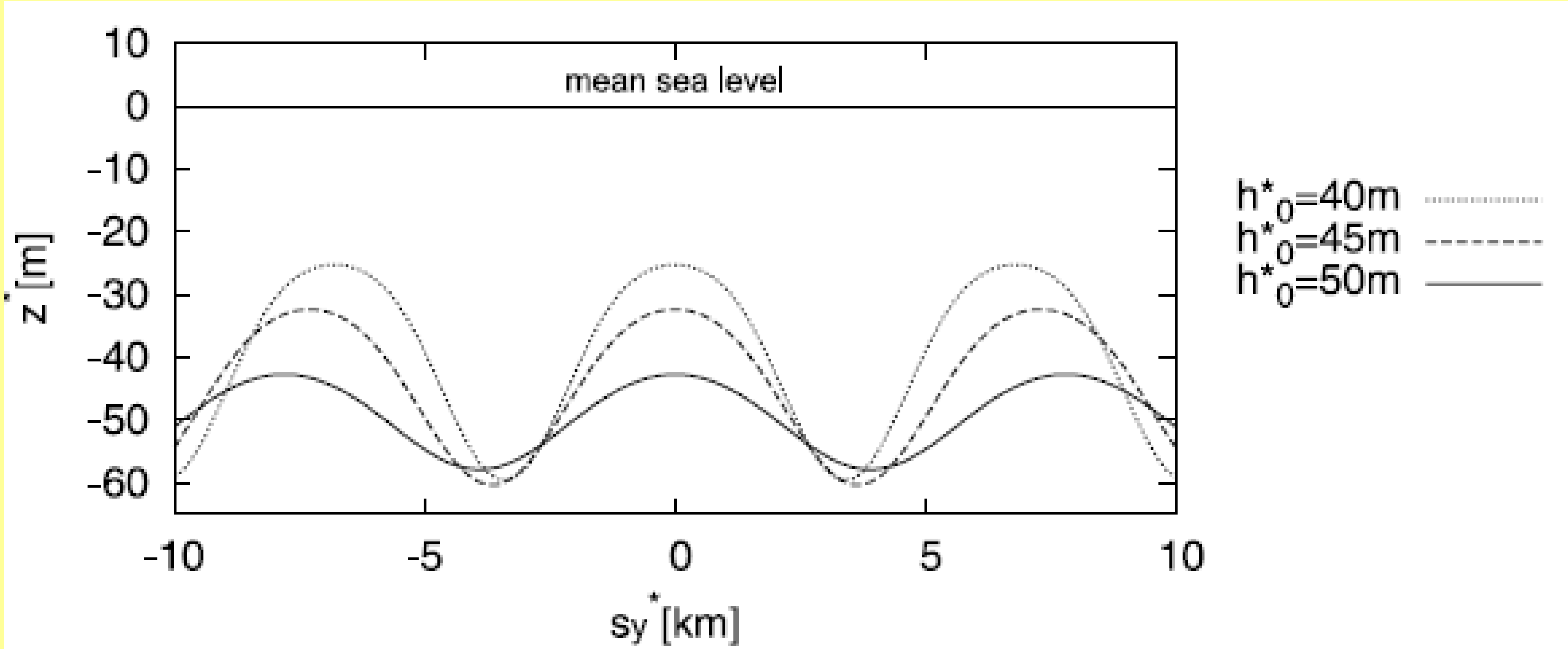
Figure 7. Equilibrium bottom profile in a plane orthogonal to the sand bank crests. The coordinate s_y^*, perpendicular to the sand bank crests, is scaled with the crest-to-crest distance L_{sb}^* of the bottom forms, while the average water depth $h_0{}^*$ is used to scale the vertical coordinate z^*. The values of the dimensionless parameters are $r = 85$, $e = -0.9$, $z_r = 6 \times 10^{-4}$, $f = 0.8$, $\varphi = 7.5°$, $d = 4 \times 10^{-6}$, and $\psi_p = 1.52 \times 10^{-2}$.



0.6
0.5
0.4
0.3
0.2
0.1
0
|A1e|
0.54
0.56
0.58
0.6
0.62
0.64
U0* [m/s]



mean sea level
0
-0.5
-1
-1.5
-2
z*/h*0
0
0.5
1
1.5
2
2.5
3
3.5
4
sy*/L*sb
r= 85
r=100
r=115
r=130



mean sea level
10
0
-10
-20
-30
-40
-50
-60
z* [m]
-10
-5
0
5
10
sy* [km]
h*0=40m
h*0=45m
h*0=50m